AN ANCIENT METHOD FOR COOLING WATER EXPLAINED BY MASS AND HEAT TRANSFER

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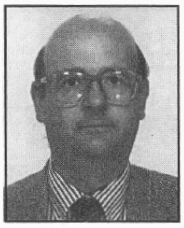
Educators know that presenting real-life examples in the classroom helps students to understand the principles in engineering education. They are always on the lookout for examples that will confirm textbook equations and principles[1-4] and for novel problems that will stimulate student interest once students have mastered the more routine skills in engineering.

In this paper we present an example in which the application of basic concepts normally introduced in the sophomore heat and material transfer class allow the quantitative explanation of an ancient method of chilling water. The experiment and exercise cover several important concepts in a variety of topics, including material and heat balances, thermodynamics, psychrometry, differential equations, and numerical methods. It shows how to put the concepts together to analyze a familiar effect.

An earthenware pitcher with a spout and a handle (called a *botijo* in Spanish) is a liquid container used for centuries in Spain and other countries to chill drinking water. Most people know that earthenware is a clay-based ceramic ware, or pottery, that has not been fired to the point of vitrification and is thus slightly porous and coarser than stoneware or porcelain.[5] It was developed by people in ancient civilizations who found that clay could be mixed with water, shaped, dried, and placed in a fire to harden.[6]

With the advent of refrigeration, the use of this pottery has diminished, but it is still used by some segments of society, such as farmers and bricklayers who often do not have a convenient source of cool drinking water. The *botijo* is a familiar object in Spain not only because it keeps water cold, but also because it provides a characteristic and enjoyable mineral flavor.



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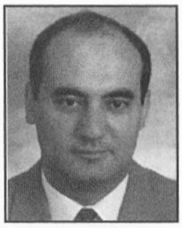

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At the secondary level or in freshman chemistry and physics courses in Spain, when concepts such as evaporation and heat transfer are studied, it is not unusual that the students are asked why water contained in a *botijo* is cooled. The answer could be that the porous ceramic material contains dead-air spaces that have a very low thermal conductivity. On the other hand, the water exudes through the pores and evaporates into the air, and the energy required to sustain the evaporation (*e.g.,* the latent heat of vaporization of the water) must come from the internal energy of the liquid, which then must experience a temperature reduction. But those answers are only a qualitative approach to this particular problem of transpiration cooling. A more quantitative answer follows.

EXPERIMENT

A summer day was simulated with an oven at 39.0°C (in this manner the external temperature can be maintained constant). The measured relative humidity in the laboratory was 42% and the temperature was 27.5°C. We poured 3.161 kg of water at 39.0°C into the *botijo* (placed previously in the oven), immersed in it a long thermometer with an accuracy of 0.1°C, and then measured the loss of water mass (due to evaporation) by removing the entire jar from the oven and weighing it (with an accuracy of 1 g) periodically. An assumption is that the removal does not disturb the experiment due to the slowness of the evaporation process and the high heat content of the water. A photograph of the experimental setup is shown in Figure 1.

We observed that the water temperature fell quickly (in about seven hours) to about 24°C, with a loss of mass of about 400 g. About three days later, after an increase in the water temperature (slow at first and abrupt near the end) the water was completely evaporated, with an end temperature of about 39°C. It should be noted at this time that the point of using this kind of jar in real life is to chill water on a warm day, and it obviously is full for only a few hours before it is drunk.

MASS AND HEAT TRANSFER MODEL

The *botijo* is modeled as a sphere of 0.10-m radius, as shown in Figure 2. The volume occupied by the water is

$$V = (4/3)\pi R^3 - (\pi/3)\left(3Rh^2 - h^3\right)$$

The water interior surface is

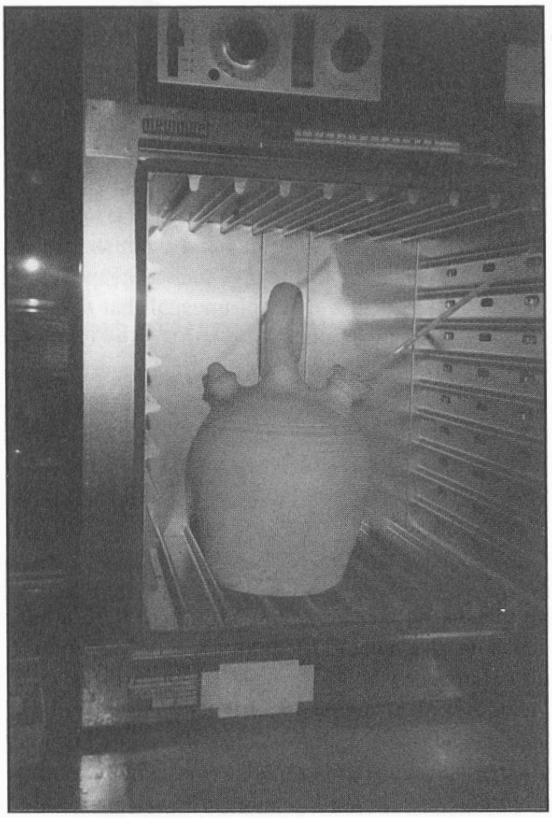

Figure 1. *Photograph of the experimental device.*

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$$A = \pi\left(2Rh - h^2\right)$$

and the wet exterior surface is

$$S = 2\pi R(2R - h)$$

Given the fact that the density of the water is around the unity, considering a V of 10^3 cm^3 is equivalent to considering the mass of water in kilograms.

In the mass and heat transfer model that has been developed, the following assumptions have been made:

1. *The earthenware pitcher is perfectly spherical.*
2. *The porous material is perfectly permeable to water and permits formation of a stable and continuous film in the wet outer surface. Thus, there is no additional resistance to the mass transfer of water.*
3. *There is no loss of water by dripping or exuding.*
4. *The mass transfer coefficient at the outer surface and the inner surface (free surface) is the same (simplifying the mathematical treatment).*
5. *The surface of the liquid at the interface with air is at constant temperature and is in equilibrium with the air.*
6. *The dry wall above the liquid is maintained at the oven temperature of 39.0 °C and radiates to the inner surface of liquid (at θ_s = 24.2 °C).*

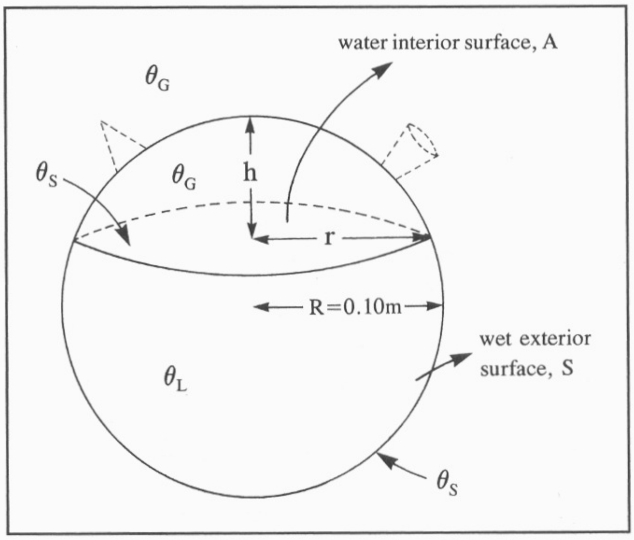

Figure 2. *Sketch of the geometrical model taken for the mass and heat transfer model where θ_G, θ_S, and θ_L are the air temperatures at the surface of the water and in the water, respectively.*

7. *The overall heat transmission coefficient U lumps together all the resistances of convection in the liquid and conduction in the liquid and through the wall.*
8. *The liquid is perfectly mixed.*
9. *The heat capacity of the pottery wall material is rather low in comparison to the heat capacity of the water. Thus the heat content of the jar is assumed negligible.*
10. *The shape factor of radiation between surfaces, f, is constant.*
11. *There is a total renovation of the air in the oven at every moment.*
12. *The humidity and temperature of the air (in the oven and at the laboratory) do not vary during the process.*
13. *The methods of measurement of mass and temperature are sufficiently quick so as not to alter the process and its results.*

According to principles of mass and heat transfer given in textbooks,[7-10] the system of differential equations describing the physical situation is

$$-\frac{dV}{dt} = k'a(H_s - H) \tag{1}$$

$$VC_p\left(\frac{d\theta_L}{dt}\right) = h_c a(\theta_G - \theta_S) + f\varepsilon\sigma\left[(273+\theta_G)^4 - (273+\theta_s)^4\right](4\pi R^2 - S) - U a(\theta_L - \theta_S) - \lambda_w\left(-\frac{dV}{dt}\right) \tag{2}$$

Equation 1 expresses the water evaporation rate as a function of the mass transfer coefficient of the water (k'), the total water surface (a=A+S) and the difference between the saturation humidity of the air (H_S) (*e.g.*, in the equilibrium with wet-bulb or adiabatic saturation temperature), and the actual air humidity (H). The values for humidities are obtained by using a psychrometric chart[9] for 1 atm and in accordance with the experimental conditions are:

H = 0.011 kg water/kg dry air

H_S = 0.018 kg water/kg dry air

By fitting the experimental evaporation data into Eq. 1 by the least-squares method, a value for k' of 80 kg water/hm^2 was obtained. There is very good agreement between this obtained coefficient and values obtained in accordance with Perry's Handbook.[11]

Equation 2 is a lumped analysis which expresses the thermal variation of the liquid and corresponds to a balance between heat transfer from the air to the water (thermal convection over the surface A+S and thermal radiation by the air in the chamber without water), the heat loss from the liquid to the interphase liquid-vapor (measured by the overall heat transmission coefficient per unit area, U) and the evaporation of liquid (measured by the latent heat of vaporization, λ_w). The specific heat capacity of water is

$$C_p = 1.0 \text{ kcal / kgK}$$

According to Sherwood and Pigford,[12] the values of h_c and k' are related by the expression

$$\frac{h_c}{k'} = s\,Le^{2/3} \tag{3}$$

where

s = wet heat of air

Le = Lewis number for the air-water system

In our case, using Le = 1.15 and s = 0.24 kcal/kgK, h_c / k' results in a value of 0.26.

The value of θ_G is 39.0°C as measured, and the value of θ_S is 24/2°C, as shown in the psychrometric chart. For the latent heat of vaporization of water we have taken the value

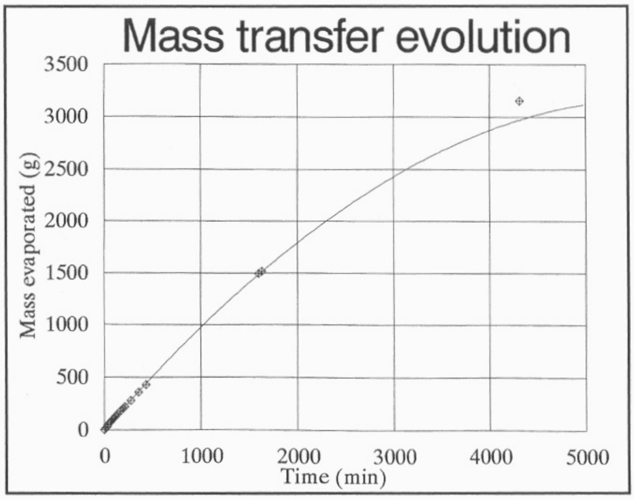

Figure 3. *The mass of water evaporated vs. time, using Eq. (4): experimental ⊞; fitted —*

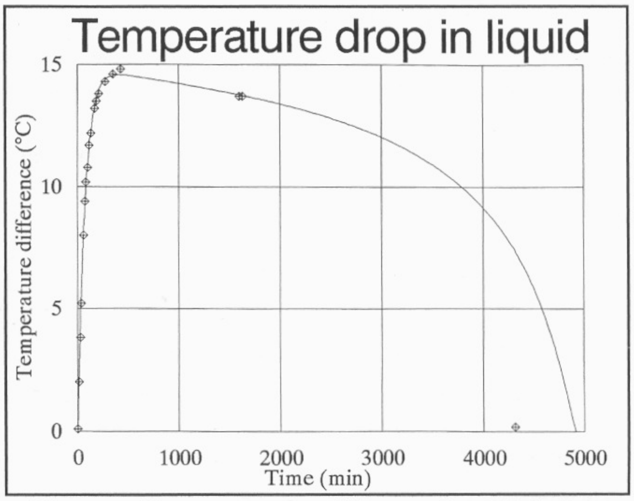

Figure 4. *The temperature difference (39.0°C-θ_L) vs. time, using Eq. (5): experimental ⊞; fitted —*

of 24.2°C, which is λ_w = 583 kcal/kg. The least-squares method gives

$$f\varepsilon\sigma = 3.05 \cdot 10^{-8} \text{ kcal/hm}^2\text{K}^4 \quad \text{and} \quad U = 22 \text{ kcal/hm}^2\text{K}$$

These two values are of typical magnitudes. The Stefan-Boltzmann constant is

$$\sigma = 4.9 \cdot 10^{-8} \text{ kcal/hm}^2\text{K}^4$$

f is the shape factor of heat radiation, and ε is the emissivity of surface. These two latter values must be in the range 0-1, as found. On the other hand, the values of U given by Perry[11] for similar systems are on the order of magnitude of that obtained by regression.

By substituting the above values, Eqs. (1) and (2) reduce, respectively, to

$$-\frac{dV}{dt} = 0.56(A+S) \tag{4}$$

$$\frac{d\theta_L}{dt} = \frac{6.41 - 51S + (A+S)(840.2 - 22\theta_L) + 583\left(\frac{dV}{dt}\right)}{V} \tag{5}$$

RESULTS AND DISCUSSION

Both differential equations, together with the simultaneous calculations of V, A, S, and h values (geometric parameters dependent only upon the volume of water in the vessel) were solved by numerical methods (algorithms of 4th-order Runge-Kutta and Newton), giving the values of the mass of evaporated water and the temperature drop of the liquid as functions of time, as represented in Figures 3 and 4.

It can be seen in the figures that the agreement between experimental data and the values obtained according to the mass and heat transfer model is very good, with the exception of the last experimental value where several assumptions (such as Nos. 1 and 9) are no longer valid.

CONCLUDING REMARKS

We believe that practical applications of the type described in this paper enhance students' understanding of the principles of heat and mass transfer. We hope that using such a novel system will also serve to make the international community aware of this ancient Spanish method of chilling water.

In developing the model it was necessary to consider the effect of radiation in heating the water in the jar. This is an interesting aspect of the experiment that was not initially expected and it fits well the experimental data.

The values of the regressed parameters obtained are reasonable in terms of their physical significance, showing the usefulness of this problem.

The number and variety of concepts used in modeling this heat and mass transfer system that has been used for hundreds of years produce an example of interdisciplinarity that distinguishes chemical engineering.

NOMENCLATURE

- a — total external surface of the water; a = A + S (m^2)
- A — water surface in contact with the air in the chamber without water (m^2)
- C_p — specific heat capacity (kcal/kg K)
- f — shape factor of heat radiation (adim.)
- h — height from the water to the top of the pottery (m)
- h_c — heat convection coefficient of air per unit area (kcal/hm²K)
- H — humidity of the air (kg/kg)
- H_S — saturation humidity (kg/kg)
- k′ — mass transfer coefficient of the water (kg/hm²)
- Le — Lewis number (adim.)
- r — radius of the external surface of the water in contact with the air (m)
- R — radius of the pottery (m)
- s — wet heat of air (kcal/kgK)
- S — external surface of the water in contact with the air (m^2)
- t — time (h)
- U — overall heat transmission coefficient of water per unit area (kcal/hm²K)
- V — volume or mass of water (kg)

Greek Symbols

- ε — emissivity of surface (adim)
- θ_G — temperature of the air (°C)
- θ_L — temperature in the inner of the water (°C)
- θ_S — temperature at the outer of the water (°C)
- λ_w — heat of vaporization of water (kcal/kg)
- σ — Stefan-Boltzmann constant (kcal/hm²K⁴)

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